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EXPERIMENTS WITH A ROTATING-CYLINDER

VISCOMETER AT HIGH SHEAR RATES

By J. A. Cole, R. E. Petersen, and H. W. Emmons

Harvard University



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SUMMARY

Two straight mineral oils and a polymer-containing oil have been tested in a rotating-cylinder viscometer at high shear rates (maximum 0.25 million reciprocal seconds) and the accompanying heat effects have been investigated.

The torque measurements are of low accuracy and fail to establish the moderately non-Newtonian behavior of the polymer-containing oil, but the temperature measurements, which are in good agreement with a thermal analysis, do indicate the presence of temporary viscosity decrease at high shear rates for this oil.

INTRODUCTION

In full fluid film or hydrodynamic lubrication the viscosity is the most important physical property of the lubricant, and other quantities involved, such as density, thermal expansion coefficient, heat capacity, and thermal conductivity, are of relatively minor concern. The load capacity and friction torque of a bearing are directly dependent on the viscosity of the lubricant, and thus the bearing performance is controlled by the influence of the operating temperature, pressure, and shear rate or stress on the viscosity.

The effect of temperature on viscosity is considerable and well known. To indicate orders of magnitude, a light mineral oil may undergo a tenfold reduction in viscosity for a temperature rise from 25° C to 100° C. The effect of pressure is less striking; but, in view of the high film pressures generated in heavily loaded bearings, it is still important and has been widely studied. A light mineral oil, for example, may increase fifteenfold in viscosity at 25° C and fivefold at 100° C for a pressure rise from 1 to 1,000 atmospheres.

Most lubricating oils are Newtonian, at any rate over the normal range of operating conditions, and viscosity is independent of shear

rate. Nevertheless, the possible effect of shear rate has been a matter of interest for many years inasmuch as there has been some controversy as to whether straight mineral oils show non-Newtonian behavior at very high shear rates, say over 1 million reciprocal seconds. With the increasing use of very high speed bearings this question has gained in significance, since in high-speed bearings the whole of the oil film is subjected to a high shear rate, whereas in heavily loaded lower speed bearings the shear rate is high only over a small film arc near the point of minimum film thickness.

It is, however, the quest for lubricants suitable for use at extremes of temperature, high or low, and/or with improved viscosity-temperature characteristics, that has caused an increase of interest in non-Newtonian effects. Synthetic additives (viscosity index improvers) for mineral oils have been developed which give a flatter viscosity-temperature characteristic attributable to the coiling of long molecular chains; that is, the temperature rise causes uncoiling due to increased solubility in the base oil and this change of form partially offsets the viscosity drop normally arising from an increase in temperature.

The effectiveness of viscosity index improvers increases, in general, with increase in molecule chain length but is in part vitiated by the effect of shear rate. Under shear conditions there can be a permanent viscosity loss due to mechanical breakdown of the molecular chain into smaller molecules, and there can be temporary or reversible viscosity loss due to molecular orientation. These effects become more pronounced with increase in molecular chain length so that a compromise must be sought between the use of small quantities of high-molecular-weight additives on the score of economy and the use of larger amounts of lower molecular weight compounds on the score of stability.

It may be mentioned that the amounts of the polymerized esters of acrylic and methacrylic acids or polymerized isobutylene used for this purpose are small, of the order of 2 or 3 percent by weight; and, since their molecular weights may be 100 times that of the base oil, the molecular ratio is very small indeed. The mechanism of temporary viscosity loss is the same as that expected for straight mineral oils, but the longer molecules suffer alinement at lower shear stresses.

An experimental study of non-Newtonian effects in lubricants is complicated by the tendency for high shear rates to be associated with considerable heat dissipation due to viscous losses, so that viscosity drops due to shear rate are overshadowed by viscosity drops due to temperature rise. Thermal effects at high shear rates can be minimized by a suitable choice of variables in both capillary and rotating-cylinder viscometers, since the rate of heat production for either is approximately proportional to the product of viscosity, film thickness, and shear rate squared. It is therefore clearly advantageous to obtain the high shear rate required as far as possible by use of small film thicknesses or capillary diameters.

In regard to the choice of equipment for this type of investigation, the rotating-cylinder instrument is generally preferred to the capillary instrument when all factors are taken into account. The latter is probably simpler and has the advantages of a static piece of equipment, but, on the other hand, the pumping necessary introduces the further complication of pressure-viscosity effects and possible external permanent viscosity loss. The shear stress and shear rate vary across the capillary cross section, whereas in the rotating-cylinder instrument shear stress is constant across the film and shear rate varies only when velocities and film thicknesses are large. With the capillary tube, the time of flow passage at high shear rates may be so small that molecular orientation may not have time to become fully established, whereas in the rotating-cylinder viscometer the same fluid sample can be continuously sheared.

The desirability of using a small film thickness in the rotating-cylinder viscometer is subject to the limitations imposed by difficulty and cost of construction. With a small clearance such matters as roundness, alinement, surface finish, and thermal expansion become increasingly important, and as small clearances are difficult to measure accurately the instrument must be calibrated in terms of viscosity instead of being used absolutely. Ultimately, when the film thickness is comparable with the amplitude of the surface roughness, there is danger that anomalous results may be obtained when hydrodynamic effects are measured on a microscopic scale.

This investigation, which was begun in 1948, covers the design, construction, and modification of an apparatus for determining hydrodynamic bearing performance and the tests made using this apparatus (refs. 1 to 4 and present report). The lubricants tested were kerosene (ref. 2), from which anomalous viscosity results were obtained, n-butyl sebacate, silicone GE 200, and Etna Oil Light (ref. 3), all of which proved to be Newtonian when tested at shear rates up to 0.224, 0.067, and 0.206 million reciprocal seconds, respectively. In the present report, which is the last phase of this investigation, thermal conditions in the viscometer are investigated at high shear rates when using both Newtonian and non-Newtonian oils of known properties. As is evident from the comparative figures in table I, in which the work of other investigators (refs. 5 to 20) is summarized, the present viscometer is inefficient in the sense that the shear-rate range is not very great while the heat dissipation is large. This difficulty can be overcome by further modification, but as it stands the equipment is very well suited to an exploratory examination of the thermal conditions which will arise at very high shear rates of the order of several million reciprocal seconds when recourse to small clearances as in the viscometers of Needs (ref. 19) and Barber, Muenger, and Villforth (ref. 20) will no longer prevent the complication of thermal effects.

The basic theory for thermal effects first developed by Bratt as discussed in reference 21 and extended by Hagg (ref. 15) is used for this study. Blok, in a paper presented at the Sixth International Congress for Applied Mechanics, has given a very useful generalization of this theory, and the results are given in appendix A. The analysis, which is based on the existence of laminar conditions in a Newtonian fluid and on the assumption of an exponential viscosity-temperature relation (valid for a not greater than fourfold variation of lubricant viscosity across the film), expresses rotor-stator temperature difference ΔT , apparent viscosity η_{ar} , and shear stress S (all expressed nondimensionally) in terms of two nondimensional variables P and N given by the following formulas:

Heat-partition factor,

$$P = \frac{\text{Heat conducted from film via stator}}{\text{Heat generated in film}}$$

Heat-dissipation factor,

$$N = \frac{\beta \eta_s U^2}{k}$$

where

- β viscosity-temperature coefficient
- η_s viscosity at stator temperature
- U surface speed
- k thermal conductivity of lubricant

The theory shows that the relative viscosity (the ratio of the apparent viscosity η_{ar} deduced from the measured torque to the viscosity η_s at the stator surface temperature T_s) decreases from the isothermal value of unity as P and N increase.

It has been necessary to extend the analysis for the present research, and these new results appear in appendix B. Expressions are given for the film temperature gradient, a quantity measured in the present report instead of rotor-stator temperature difference, and for the temperature correction to the stator surface temperature to bring it into line with the apparent viscosity. The latter is a more exact form of the "parabolic" constant-viscosity correction used earlier in

this investigation (ref. 3), and it is compared with a generalized form of this parabolic correction. Since the rotating-cylinder viscometer is a constant shear-stress instrument, the analysis can be extended to non-Newtonian fluids provided that the ratio of the high-shear to zero-shear viscosity is a function of the shear stress only. An examination of existing information on non-Newtonian lubricants suggests that this is not an unreasonable assumption. These new results permit the effects of temperature and shear stress on viscosity to be separated.

The measurement of viscosity by flow and pressure drop through the annulus necessitates an extension of the theory to account for the effect of the decrease in viscosity on the axial velocity. The pressure drops used were so small that the axial shear stress and the flow energy dissipated are both negligible compared with the corresponding rotational quantities. The results of this analysis are again expressed as a relative viscosity (the ratio of the apparent viscosity η_{aa} deduced from the flow and pressure drop to the viscosity η_s at the stator surface temperature T_s).

This work was done at Harvard University under the sponsorship and with the financial assistance of the National Advisory Committee for Aeronautics.

SURVEY OF LITERATURE

A number of writers have speculated that the possible reduction of liquid viscosity under shearing flow conditions is caused by orientation of rod-shaped molecules. Bondi (ref. 22) has calculated that for the molecular-weight range appropriate to ordinary lubricating oils, a minimum shear stress of the order of 500,000 dynes/sq cm would be necessary to produce a perceptible temporary viscosity decrease. For much larger molecules, such as the viscosity index improvers already mentioned, smaller shear stresses would be required.

It is instructive to examine earlier experimental work on flow orientation with this value in mind, amplifying a similar review presented by Blok at the Sixth International Congress for Applied Mechanics. Table I summarizes investigations carried out during the past 20 years, the earlier work dealing only with ordinary lubricants and the later work, with polymer-containing oils also. It seems that in no case was Bondi's critical shear stress reached, and, in the five cases where non-Newtonian effects were claimed for ordinary lubricants, there were also large thermal effects. The majority of the evidence indicates that straight mineral oils behave in practice as Newtonian fluids; even at shear rates as high as 2 million reciprocal seconds, the stress is unlikely to approach Bondi's critical value because of heat dissipation and consequent viscosity

reduction. The only recent investigators to claim non-Newtonian effects are Ward, Neale, and Bilton (ref. 17), in whose apparatus care was taken to measure the large thermal effects. There are two possible sources of error in their work: Heat loss via the rotor, said to be "virtually thermally insulated," would permit a temperature maximum to occur inside the film instead of at the rotor as assumed and their "critical test" for anomalous viscosity then breaks down; and, further, it seems likely that insufficient time was allowed for the establishment of thermal equilibrium.

The definitive papers on polymer-containing oils are those of Fenske, Klaus, and Dannenbrink (ref. 18), Needs (ref. 19), and Barber, Muenger, and Villforth (ref. 20) in which agreement on the extent of viscosity drop for a given oil is within about 5 percent.

SYMBOLS

h	film thickness
k	thermal conductivity
N	heat-dissipation factor, $\frac{\beta\eta_s}{k}U^2$
N'	modified heat-dissipation factor, $\frac{\beta\eta_s'}{k}U^2$
n	maximum rotational speed
P	heat-partition factor, $\frac{\text{Heat lost via stator}}{\text{Heat generated in film}}$
p	pressure
S	circumferential shear stress
T	temperature
U	rotor surface velocity
u	radial velocity
w	axial velocity
W	axial flow per unit circumference
y	film-thickness coordinate measured from rotor radially outward

β	viscosity-temperature coefficient, $\eta = \eta_s e^{\beta T}$ with $T_s = 0$
Γ	nondimensional shear stress, $Sh \sqrt{\frac{\beta}{\eta_s k}}$
ΔT	temperature difference between T_s and effective film temperature
η	absolute viscosity
η_{ar}	measured or apparent viscosity from torque measurements
η_{aa}	measured or apparent viscosity from flow and pressure-drop measurements
η_s	viscosity at stator temperature under zero-shear conditions
η'_s	viscosity at stator temperature under prevailing shear conditions
ν	kinematic viscosity

Subscripts:

r	rotor
s	stator

APPARATUS

The Harvard rotating-cylinder viscometer has been described in previous reports of this investigation (refs. 1 to 3). Briefly, it consists of a 2-inch-diameter hardened Nitralloy steel rotor supported vertically at the upper end by two angular contact precision ball bearings and rotating inside a bearing or stator of the same material, 4 inches long with 0.003-inch radial clearance. This stator is carried on a dog-leg spring mounting, whose effective torsional stiffness is approximately 0.032 pound per thousandth of an inch as measured at the stator outside diameter of 6 inches.

The rotor is driven by an electronically speed-controlled 15 horsepower direct-current motor through a gearbox and final belt drive.

Interchangeable gears and two supply voltages give a controlled speed range of 12 to 12,500 rpm and a maximum possible speed of 25,000 rpm. Motor and rotor speeds are electrically indicated.

The stator temperature field is deduced from the readings of 75 thermocouples symmetrically disposed throughout the stator and switched to a manually balanced precision potentiometer. A single thermocouple records oil inlet temperature and others record oil outlet temperatures.

Torque can be deduced from the angular deflection of the stator on its mounting, which may be indicated on an extended scale using an electronic pickup described in detail in previous reports (refs. 1 to 3) of this investigation. Since the completion of reference 4, torque measurement has been directly made by a null method, the torsion mounting being used merely as a spring pivot. Torque is applied by a dead loading system to restore the stator to the zero position as indicated by an X75 microscope. As described in a subsequent section, torque measurement is not very precise. This is ascribed to the imperfect elastic properties of the stator thermocouple leads contributing about one-third of the torsion-mounting-stiffness value quoted.

The test lubricant is supplied via a tube passing down the center of the hollow rotor. A stationary disk at the end of the tube insures that the stator measured torque does not include that due to the end of the shaft. As noted in the previous report (ref. 4), the present large clearance tends to empty and give an incomplete film unless a continuous supply of oil under a small head is arranged. In order to keep the axial flow to a minimum, an alternative oil supply system may be used, oil dripping into a constant-head sight glass attached to the stator. This does not involve a force on the stator which would affect the torque reading.

A few tests were made at several different heads. The oil flow was measured by collecting the oil in a calibrated cylinder for a measured period of time. An independent measure of viscosity was thus obtained.

The rotor-stator film resistance and capacity may be recorded during a test run.

A detailed critical examination of the design and construction of the apparatus is presented in reference 4. It was considered that the located rotor, a fundamental feature of the design, was on the whole less satisfactory than the more usual self-centered rotor employed in other designs and unsuitable for use as a preset-eccentricity journal-bearing machine. A number of less sweeping changes were suggested, but time has not permitted their adoption. These included the use of a coaxial flexible coupling for the final rotor drive in place of the present belt drive,

the use of a smaller rotor clearance, a stiffening of the viscometer framework, and the provision of rotor thermocouples.

An unexpected limitation on the maximum speed at which the viscometer could be operated arose from overheating of the main bearings and impending seizure of the fiber retainers. This difficulty increased during the course of the investigation, permitting speeds up to only 5,000 rpm, and replacement bearings were not received in time to establish whether this effect was inherent in the design, because of the inevitable high temperatures at high shear rates and absence of continuous lubrication of the main bearings, or whether it was merely a result of bearing deterioration over the several years the machine has been in use. The bearings are not protected against the ingress of dust from the atmosphere.

EXPERIMENTAL METHOD

The basic requirement for the use of the instrument under the severe thermal conditions now present at high shear rates is that temperature equilibrium shall be established, inasmuch as the thermal theory used postulates thermal equilibrium.

Torque and temperature readings during a warming-up run have shown that the rates of temperature rise and torque decrease become small after 2 to 3 hours, but experience has indicated that it is desirable to operate at the test speed for about 6 hours before making the final temperature record, and this restricts tests to one per day. Once conditions are sufficiently steady, the readings from the 75 stator thermocouples and other temperatures are successively recorded and divided for convenience into three groups. At the beginning and end of each group a control temperature is read as a check on the temperature equilibrium, the potentiometer is checked, and readings are taken of speed, torque, and oil outflow. The complete set of readings takes about 1 hour.

Stator temperatures are plotted on a large scale against the logarithm of the radius and are found to follow closely the expected linear relation. The graphical method helps to eliminate random errors and is used to give the best approximation to the extrapolated stator inside surface temperature and radial temperature gradient. The stator temperature and the gradient vary somewhat both axially and circumferentially, especially the former, and the arithmetic means are taken. Torque and speed are averaged over the whole run.

For a given oil, the surface temperature, temperature gradient, and torque should give a smooth curve when plotted against speed, with a small scatter due to day-to-day variation of room temperature. This has

been found to be a useful check on the consistency of readings, particularly when there is any doubt about temperature equilibrium or oil film completeness.

PRECISION

Thermocouple voltages are actually read to 0.1 microvolt and are probably repeatable to ± 1 microvolt. The International Critical Tables calibration for the copper-constantan thermocouples is used; the 100° C fixed point is occasionally checked and, as mentioned, the potentiometer is very frequently checked. The maximum probable errors in the extrapolated stator surface temperature and the stator gradient are estimated to be 0.05° C (maximum value, 150° C) and 0.05° C per inch (maximum value, 5° C per inch).

Low shear viscosities are measured in A.S.T.M. U-tube viscometers; temperatures are measured with the test-rig thermocouple equipment. Densities are measured in a relative-density bottle. The final centipoise viscosity is estimated to be within 1/2 percent of the true value.

The rotor speed varies 1 percent during a test run; the accuracy of measurement of speed is about 1 percent.

If the null method for torque measurement is used, there appears to be a variation in zero position due to temperature and other factors of about 60,000 dyne-cm and the microscope permits adjustment under testing conditions to about 20,000 dyne-cm. Thus, the torque error can be as high as 80,000 dyne-cm, and for the usual torque values of approximately $2\frac{1}{2}$ million dyne-cm, this represents a precision of 3 percent.

The manufacturing and measuring tolerances for the rotor and stator are ± 0.0001 inch so that the radial clearance may be at worst 0.0001 inch or 3.3 percent different from the nominal value of 0.003. The maximum possible error due to misalignment and differential thermal expansion is about 1 percent. Thus, the clearance precision is about 4.3 percent to which should be added the error due to surface curvature introduced by use of the formula

$$\text{Apparent viscosity} = \frac{(\text{Stress})(\text{Radial clearance})}{\text{Surface velocity}}$$

which is about 0.3 percent maximum.

Thus, used absolutely the viscometer may indicate apparent viscosities with errors as large as 9 percent at worst, and used relatively with a standard viscosity fluid, about half this amount.

RESULTS

During the present investigation three lubricants have been tested in the shear rate range 0.001 to 0.25 million reciprocal second. These are Etna Oil Light (Socony-Vacuum Oil Co.), an SAE 10 straight mineral oil denoted by EOL; Flowrex 300 (Socony-Vacuum Oil Co.), an SAE 30 straight mineral oil denoted by F300; API 103 (Atlantic Refining Co.), a mineral oil containing a relatively low molecular weight polymer as a viscosity index improver. Data for the API 103 non-Newtonian lubricant have already been obtained by other investigators (refs. 18 to 20).

Figures 1, 2, and 3 show the measured viscosities for each of the oils plotted on A.S.T.M. charts against effective film temperature, with the parabolic correction for the stator temperature being used. The low-shear-viscosity values obtained with U-tube viscometers and the ranges of shear rate and stress are shown.

Figures 4, 5, and 6 show the relative viscosities and indicate more clearly the extent of deviations from the expected values.

Figures 7, 8, and 9 show the film temperature gradients; a steel-oil conductivity ratio of 320 is assumed, as used previously. The thermal conductivity of all three oils has been taken as 0.00033 cal/°C units. The viscosity-temperature coefficient is taken from a graph of viscosity against temperature deduced for each oil from its low shear viscosity-temperature curve, since the coefficient varies considerably over the temperature range of these experiments even though it is sufficiently constant for the range of temperatures occurring in the film in any one test to satisfy the requirements of the analysis. Calculation of the Reynolds number shows that the maximum value in the present tests is about two-thirds of the Taylor critical value.

Measurements have been made of film dielectric constant and film electrical resistivity (the latter under a gradient of 0.6 kilovolts direct-current per centimeter). For both the Newtonian and the non-Newtonian oils there was no significant decrease of either quantity with shear rate. Resistivity showed some decrease due to temperature rise. The measurements were of limited precision because of electrical noise at the rotor grounding brush.

In all the tests reported herein a small axial flow of lubricant was maintained to insure complete filling of the clearance space; however,

the convective heat dissipation by this flow was small and should not affect the application of the thermal analysis of appendix A. A small pumping action at high speeds, attributed to centrifugal action, has been observed and has been discussed in reference 4.

In the few tests made with varying head, in which the "axial" viscosity was obtained, the dissipation remained negligible.

DISCUSSION

In the A.S.T.M. viscosity-temperature charts of figures 1, 2, and 3, the random deviations of the measured viscosities from the mean straight lines drawn and the displacements of these lines from the low-shear lines are in general agreement with the estimate of precision given previously.

The disparity between the observed and low-shear viscosities for EOL is larger than that previously reported by Emmons and Soo (ref. 3) using the same apparatus. Their low-shear viscosities appear to be incorrect however; this is probably attributable to their use of a Saybolt viscometer.

The low accuracy of viscosity measurement is emphasized by the relative-viscosity—heat-dissipation-factor curves of figures 4, 5, and 6 in which are also indicated the expected deviations from the low-shear viscosity ratio of unity due to film temperature rise and, in the case of API 103, due to non-Newtonian behavior. The latter values are based on results by Needs (ref. 19) for this oil, and figure 5 brings out the constant-stress characteristic of the viscometer. For a given oil, there is a comparatively small change in shear stress over a range of shear rate, and the non-Newtonian viscosity decrease for API 103 is fairly constant at only 5 or 6 percent over most of the shear-rate range covered. This oil contains a low-molecular-weight polymer and is not markedly non-Newtonian. The present equipment fails to establish any non-Newtonian behavior according to figure 5.

Figure 6(b) shows the three test points obtained from axial-flow measurements. The data accuracy is insufficient to establish the correctness of the analysis.

Figures 7, 8, and 9 show the film temperature gradient at the stator in terms of the heat-dissipation factor. In the case of API 103 (fig. 9), the factor is the modified variable N' based on the viscosity at the stator temperature and the prevailing shear stress, Needs' results again being used, so that the theory still applies as shown in appendix B.

Values of the heat-partition factor P are indicated (fig. 9) and the agreement with the predictions of the thermal theory is good. In the case of API 103, this may be taken as confirming the values obtained by Needs for the extent of the non-Newtonian behavior of the oil, although it would be more convincing if the originally calibrated heat-partition factor P , equal to A , still applied. However, continued deterioration of the main bearings has altered P , the reason being that the increased frictional heat dissipation at these bearings at the top of the rotor reduces the temperature gradient from bottom to top and thus more of the heat dissipated in the film must escape via the stator. The two anomalous points marked A in figure 9 correspond to those similarly marked in figure 3. Excessive heat dissipation at the main bearings occurred during these tests.

CONCLUDING REMARKS

The results have confirmed the fact that the viscometer used comparatively indicates apparent viscosity with rather low precision, about ± 5 percent, and thus fails to differentiate between straight mineral oils, almost certainly Newtonian over the present range of test conditions, and a polymer-containing oil of known small non-Newtonian characteristics. The temperature measurements do permit this distinction to be made.

The thermal conditions existing in the apparatus have been thoroughly examined and are substantially as predicted by the theory, but a progressive deterioration in the main bearings has caused a change in the heat-partition calibration of the viscometer, as well as further limiting the shear-rate range available.

As it stands, the viscometer is relatively inefficient: The large clearance necessitates high speeds and, consequently, both large thermal corrections and a low-shear-stress range for an only moderate shear-rate range. Torque accuracy is low because of the thermocouple connections to the stator. Axial flow is necessary to maintain a complete film and, consequently, a fairly large test sample of lubricant is needed.

Without change, other than improved main bearings, the apparatus will be useful for an investigation of more viscous lubricants at moderately high shear rates, since larger torques can be measured more accurately. The good temperature instrumentation, improved earlier in this investigation by the addition of rotor thermocouples, makes the equipment useful for an investigation of heat effects in oil films along the lines now started. Two problems meriting attention are the effect of larger axial flows and consequent heat convection, and the heat effects under the turbulent conditions of the Taylor vortex motion, with and without axial flow.

With a change to a smaller clearance, the located-rotor feature of the design will become more objectionable, but the shear stress and rate ranges will be increased. The viscometer used in this investigation would still be inferior to the instruments of Needs and of Barber, Muenger, and Villforth because of low torque accuracy except at very high shear rates where the reappearance of large thermal effects despite the use of a small clearance would justify the thermocouple system.

Harvard University,

Cambridge, Mass., June 12, 1953.

APPENDIX A

THERMAL EFFECTS IN A BEARING OIL FILM

The results are given herein of a generalization by Blok presented at the Sixth International Congress for Applied Mechanics of Hagg's analysis of thermal conditions in a concentric bearing, the analysis being based on the following assumptions and basic equations:

(1) The bearing is infinitely long, and the film thickness h is small relative to the diameter

(2) The lubricant is Newtonian and the flow laminar

(3) Lubricant thermal conductivity k is constant, and thermal equilibrium exists

(4) Lubricant viscosity η varies exponentially with temperature T

(5) The shear stress and energy dissipation resulting from the axial flow of oil are negligible compared with the corresponding rotational effects

$$S = \eta \frac{du}{dy} \quad (\text{Newtonian flow}) \quad (1)$$

$$\eta \left(\frac{du}{dy} \right)^2 + k \left(\frac{d^2 T}{dy^2} \right) = 0 \quad (\text{Thermal equilibrium}) \quad (2)$$

$$\eta = \eta_s e^{-\beta T} \quad (\text{Viscosity is } \eta_s \text{ at stator,} \quad (3)$$

where temperature datum is taken)

In the analysis a parameter P , the heat-partition factor, and a non-dimensional variable N , the heat-dissipation factor, are introduced as

$$P = \frac{\text{Heat conducted from film via bearing stator}}{\text{Heat generated in film}}$$

$$N = \frac{\beta \eta_s}{k} U^2$$

where

U rotor surface velocity

The results are expressed nondimensionally as

$$\beta(T_r - T_s) = \log_e \left[1 + \left(P - \frac{1}{2} \right) N \right] \quad (4)$$

$$\Gamma \equiv \text{Sh} \sqrt{\frac{\beta}{\eta_s k}} = \frac{2}{\sqrt{P^2 N + 2}} \tanh^{-1} \frac{\sqrt{P^2 N^2 + 2N}}{PN + 2} \quad (5)$$

$$\frac{\eta_{ar}}{\eta_s} = \frac{2}{\sqrt{P^2 N^2 + 2N}} \tanh^{-1} \frac{\sqrt{P^2 N^2 + 2N}}{PN + 2} \quad (6)$$

which represent rotor-stator temperature difference, shear stress, and relative viscosity, respectively. Relative viscosity is the ratio of the measured or apparent viscosity to the stator temperature viscosity and is unity in the absence of thermal effects.

APPENDIX B

EXTENSION OF ANALYSIS FOR PRESENT RESEARCH

The film temperature gradient at the stator is measured in the present apparatus instead of that given in equation (4) and it can be shown to be

$$\beta \left(\frac{dT}{dy/dh} \right)_s = \frac{2PN}{\sqrt{P^2N^2 + 2N}} \tanh^{-1} \frac{\sqrt{P^2N^2 + 2N}}{PN + 2} = \frac{\eta_{ar}}{\eta_s} PN \quad (7)$$

Thus, once the apparatus has been calibrated to determine P , ideally, no further torque measurements are needed to find the relative viscosity inasmuch as it can be deduced from temperature-gradient measurements.

To obtain the effective film temperature a correction ΔT must be added to the measured stator temperature T_s in order to correspond to the measured viscosity η_{ar} and may be derived as

$$\beta \Delta T = \log_e \left(\frac{1}{2} \sqrt{P^2N^2 + 2N} \coth^{-1} \frac{\sqrt{P^2N^2 + 2N}}{PN + 2} \right) = \log_e \frac{PN}{\beta \left(\frac{dT}{dy/dh} \right)_s} \quad (8)$$

For a value of P not exceeding unity and a value of N which is small compared with unity, equations (4) to (8) can be represented approximately by

$$\beta (T_r - T_s) = \left(P - \frac{1}{2} \right) N \quad (9)$$

$$\Gamma = N \quad (10)$$

$$\frac{\eta_{ar}}{\eta_s} = 1 - \frac{1}{2} PN \quad (11)$$

$$\beta \left(\frac{dT}{dy/dh} \right)_s = PN \quad (12)$$

$$\beta \Delta T = \frac{1}{2} PN \quad (13)$$

It follows from equation (9) that for very small values of N (and therefore constant viscosity) the temperature distribution across the film is parabolic. On this assumption, the temperature correction to T_s can be shown to be

$$\Delta T = \frac{1 - 3P}{6P} \left(\frac{dT}{dy/dh} \right)_s = \frac{3P - 2}{6(P - 1)} \left(\frac{dT}{dy/dh} \right)_r \quad (14)$$

with the temperature gradient positive in the direction of y increasing from the rotor.

The case $P = 1/2$ is equivalent to the correction used in reference 4, but a higher value $P = 3/4$ has now been shown to apply to the apparatus. The parabolic correction is then adequate for all practical purposes up to $N = 0.1$.

The analysis may be extended to non-Newtonian fluids. There is theoretical and experimental evidence that the high-shear viscosity of a polymer-containing lubricant is a function of the shear stress S and may be expressed as

$$\frac{\eta_{\text{high shear}}}{\eta_{\text{zero shear}}} = f(S) \quad (15)$$

where $f(S)$ is a nondimensional function to be determined (probably of form $e^{-\gamma S}$ (γ constant)).

If N is still defined as $\frac{\beta \eta_s}{k} U^2$, where η_s is still the standard zero-shear viscosity at the stator temperature, a modified variable $N' = \frac{\beta \eta_s'}{k} U^2$ can be introduced, where η_s' is the viscosity at the prevailing shear stress and the stator temperature.

Analysis then shows that

$$\beta \left(\frac{dT}{dy/dh} \right)_s = \frac{2PN'}{\sqrt{P^2(N')^2 + 2N'}} \tanh^{-1} \frac{\sqrt{P^2(N')^2 + 2N'}}{PN' + 2} \quad (16)$$

$$\frac{\eta_{ar}}{\eta_s} = \frac{2}{\sqrt{P^2(N')^2 + 2N'}} \tanh^{-1} \frac{\sqrt{P^2(N')^2 + 2N'}}{PN' + 2} \quad (17)$$

For a fluid of unknown shear properties, measurement of the temperature gradient enables N' to be deduced from equation (16) and hence

$\frac{\eta_{ar}}{\eta_s}$ can be found from equation (17). The ratio $\frac{\eta_{ar}}{\eta_s}$ is known from the measurements so that the value of $f(S)$ appropriate to the shear stress prevailing during the measurements can be deduced as follows:

$$f(S) = \frac{\eta_{ar}/\eta_s}{\eta_{ar}/\eta_s'} \equiv \frac{N'}{N} \quad (18)$$

These results, believed to be new, should be of service in separating viscosity decreases due to thermal effects and to shear-stress effects during tests on non-Newtonian fluids at high shear and thermal conditions.

In order to determine axial flow, the assumptions as stated in appendix A are accepted. However, the following additional basic equations are required:

$$\frac{dp}{dz} + \frac{d}{dy} \eta \frac{dw}{dy} = 0 \quad (\text{axial momentum equation}) \quad (19)$$

$$W = \int_0^h w \, dy \quad (\text{continuity}) \quad (20)$$

The integration of equation (19) and the introduction of $\eta = \frac{S}{du/dy}$ gives

$$S \frac{dw}{du} = - \frac{dp}{dz} y + A \quad (21)$$

where A is the constant of integration. Now, integrate with respect to u from the rotor ($u = U$, $w = 0$) to the stator ($u = 0$, $w = 0$):

$$Sw = - \frac{dp}{dz} \left(\int_0^u y \, du - \frac{y}{U} \int_0^U y \, du \right) \quad (22)$$

Substitution of equation (22) into equation (20) results in the following relation between the flow per unit of circumference W and the pressure gradient:

$$SW = -\frac{dp}{dz} \left[\int_0^h dy \int_0^u y du - \int_0^h \frac{u}{V} dy \left(\int_0^U y du \right) \right] \quad (23)$$

Notice that this relation between the axial flow and the pressure drop is independent of the viscosity and its variation (with temperature and rate of shear) within the original assumptions.

For viscosity as given by equation (3), Blok gives the relation between tangential velocity u and the position y as follows:

$$\frac{h-y}{h} = \frac{\tanh^{-1} \left[\frac{(P^2 + 2/N)^{1/2}}{P + \frac{2}{N} \frac{U}{u}} \right]}{\tanh^{-1} \left[\frac{(P^2 + 2/N)^{1/2}}{P + 2/N} \right]} \quad (24)$$

By expansion of u as a power series in y , the integrals in equation (23) can be evaluated, and a final result may be obtained as

$$W = -\frac{dp}{dz} \frac{h^3}{12\eta_s} \left[1 + N \left(-\frac{1}{5} + \frac{7}{2} P - 3P^2 \right) - N^2 \left(\frac{7}{60} + \frac{17}{12} P + \frac{102}{420} P^2 + \frac{P^3}{8} \right) + \dots \right] \quad (25)$$

In this formula the expression within brackets is to be identified with η_s/η_{sa} . The following approximate formula is then obtained:

$$\frac{\eta_{sa}}{\eta_s} = 1 - N \left(-\frac{1}{5} + \frac{7}{2} P - 3P^2 \right) \quad (26)$$

This result is plotted in figure 6(b).

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TABLE I
ANALYSIS OF PUBLISHED INVESTIGATIONS OF NON-NEWTONIAN
BEHAVIOR IN LUBRICATING OILS

Reference	Type of viscometer	h, in.	n, rpm	$\frac{du}{dy}$, million reciprocal sec	δ , dynes/sq cm (a)	Thermal effects	Non-Newtonian effects observed in -	
							Straight mineral oils	Polymer- containing oils
5	Tapered rotating cylinder	0.0008		0.06	-----	Large	Yes	
6	Tapered rotating cylinder	.001	1,500	.2	3.1×10^5	Large	No	
7	Capillary	.004		1.5	4.9	Large	No	
8	Rotating cylinder and tapered rotating cylinder	.0033	4,000	.3	.1	Small	No	
9	Capillary	.005		.16	4.0	Large	No	
10	Rotating cylinder	.001	1,000	.14	-----	Small	No	Yes
11	Rotating cylinder	.0007	3,500	1.0	-----	Large	Yes	
12	Rotating cylinder	b.08	700	b.001	~.07	Large	Yes	
13	Capillary	-----		-----	-----	Large	No	
14	Rotating cylinder	-----		-----	.2	Large	No	Yes
15	Tapered rotating cylinder	.00055	16,500	2.0	.6	Large	No	
16	Capillary	c.004		c.1	c.6		c.No	c.Yes
				d.28	d.5		d.Yes	
17	Tapered rotating cylinder	.001	5,000	.3	.15	Large	Yes	Yes
18	Capillary	.01		1.0	.15	Large	No	Yes
19	Tapered rotating cylinder	.00015	600	1.8	.5	Small	No	Yes
20	Rotating cylinder	.00012	1,150	.5	1.0	Small	No	Yes
1 to 4 and present report	Rotating cylinder	.003	12,500	.43	~.08	Large		

^a δ depends on viscosity and does not necessarily occur at maximum shear rate $\frac{du}{dy}$. Bondi's (ref. 22) critical value is 5.

^bQuestionable value.

^c1946.

^d1951.

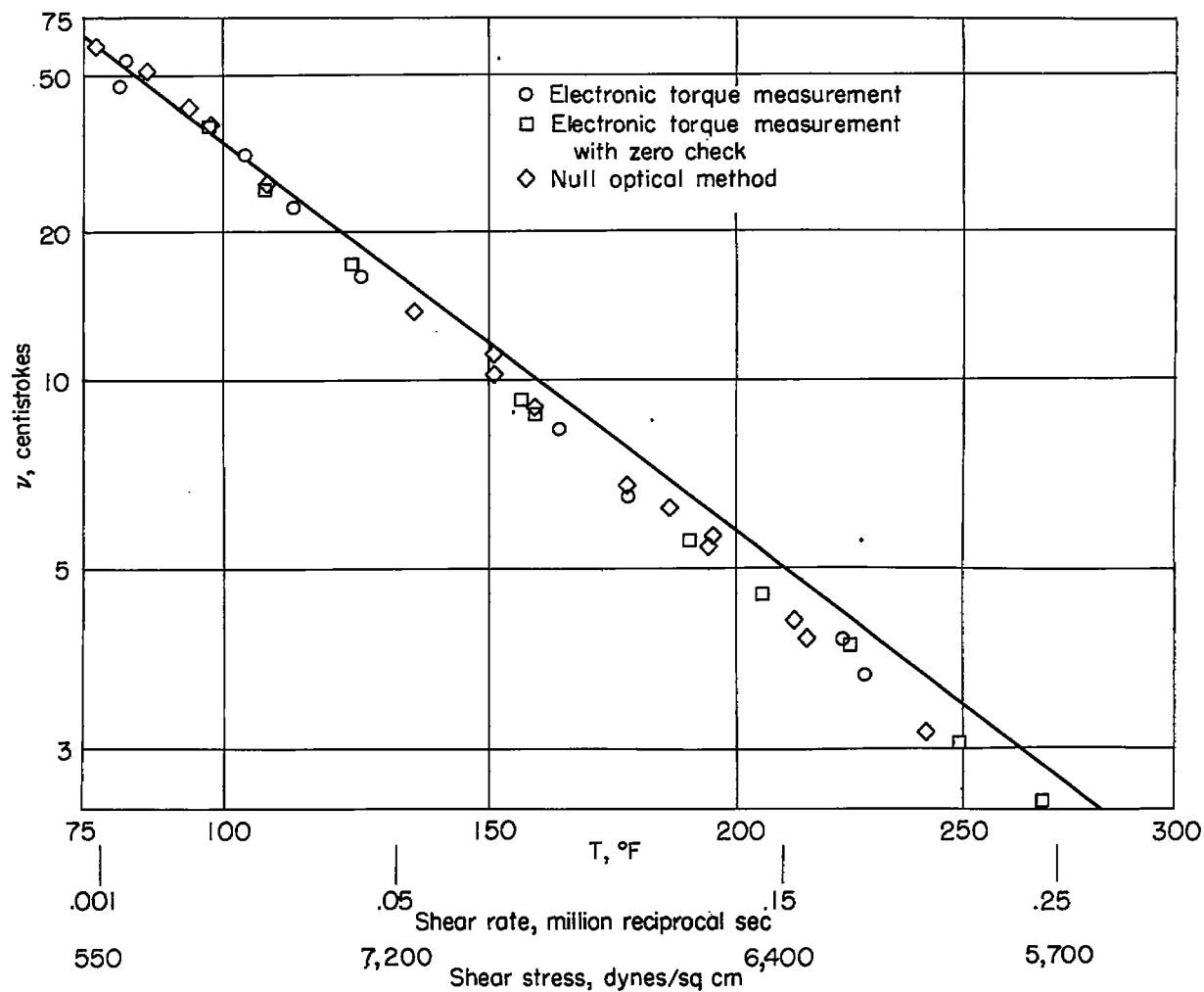


Figure 1.- Kinematic-viscosity—temperature results for Etna Oil Light. Continuous line represents standard low-shear values. Subsidiary scales indicate approximately shear rate and shear stress. Shear-stress scale has a maximum.

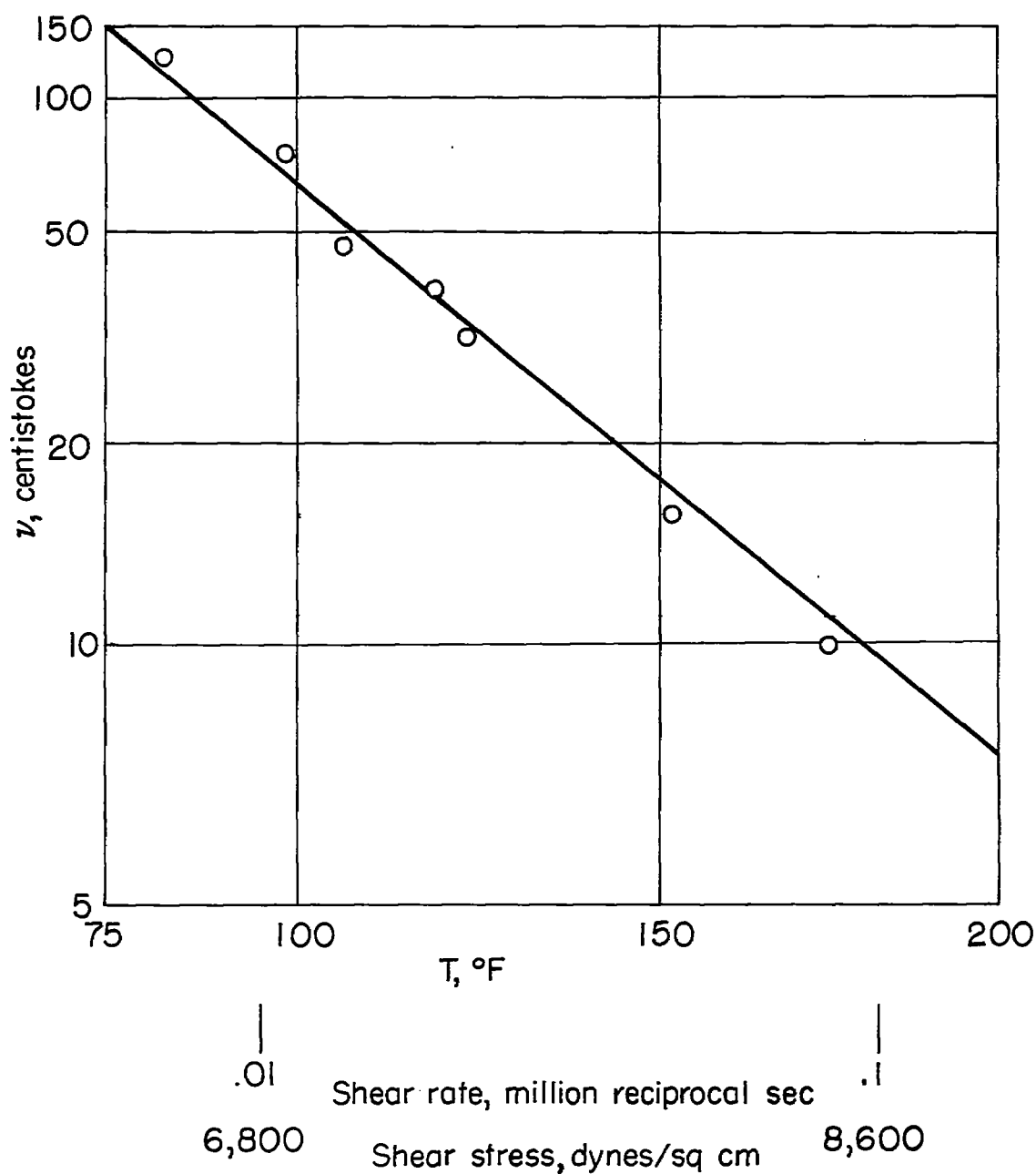


Figure 2.- Kinematic-viscosity-temperature results for Flowrex 300. Continuous line represents standard low-shear values. Subsidiary scales indicate approximately shear rate and shear stress.

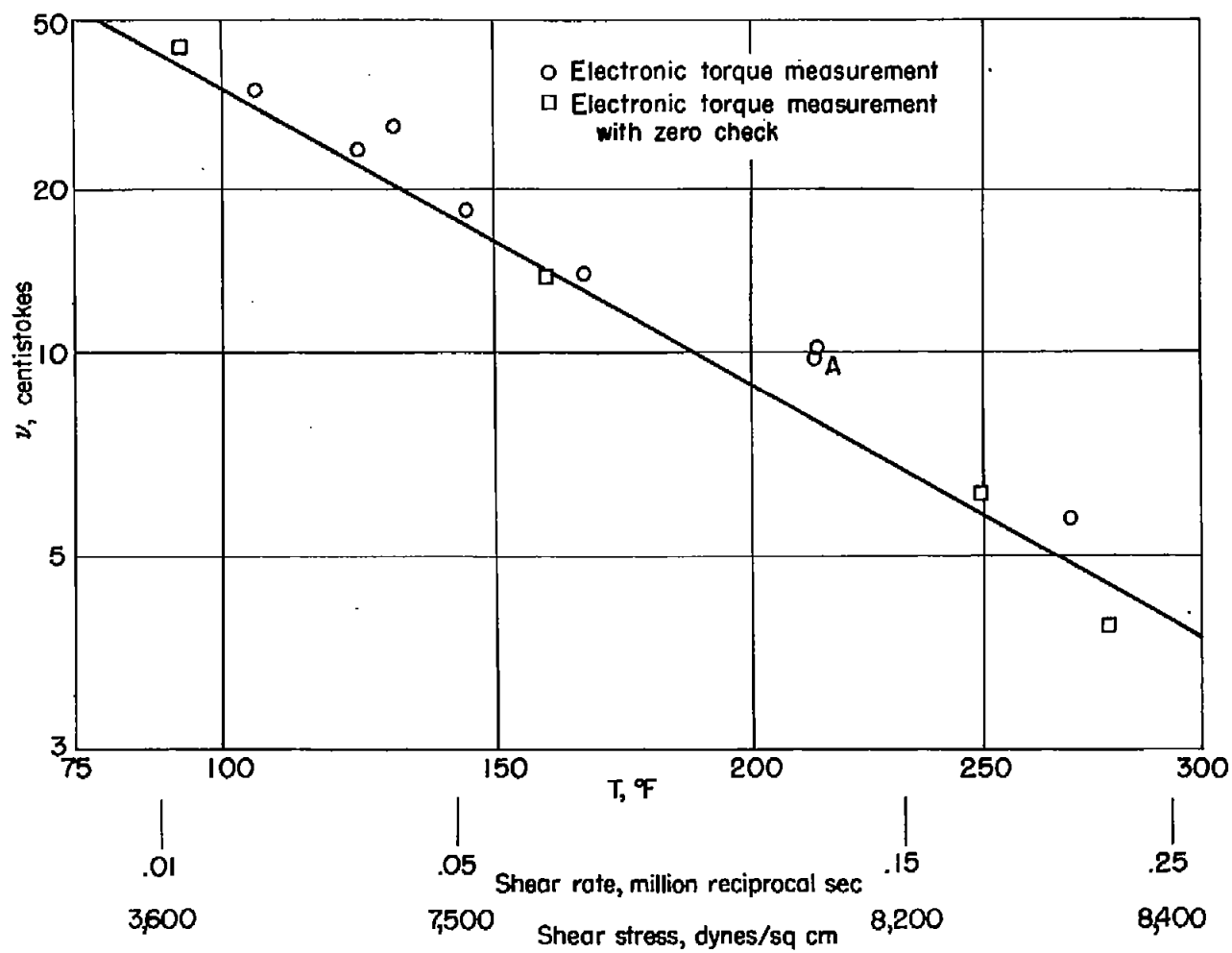


Figure 3.- Kinematic-viscosity--temperature results for API 103. Continuous line represents standard low-shear values. Subsidiary scales indicate approximately shear rate and shear stress. Points labeled A indicate anomalous values.

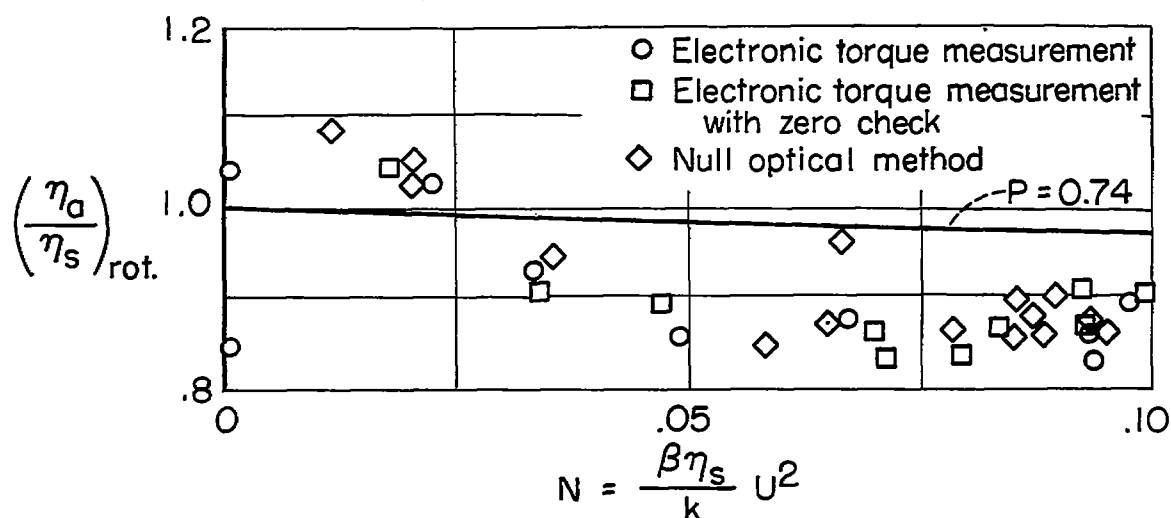


Figure 4.- Relative viscosity results for Etna Oil Light. Continuous line ($P = 0.74$) based on theory in appendix A.

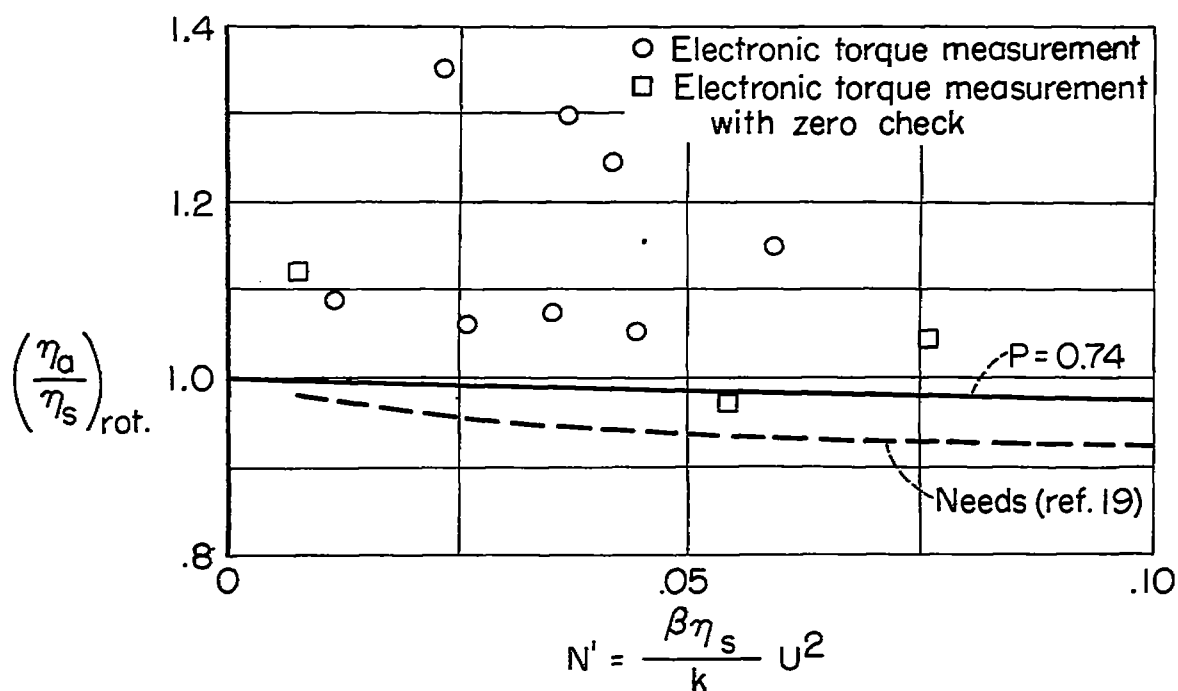
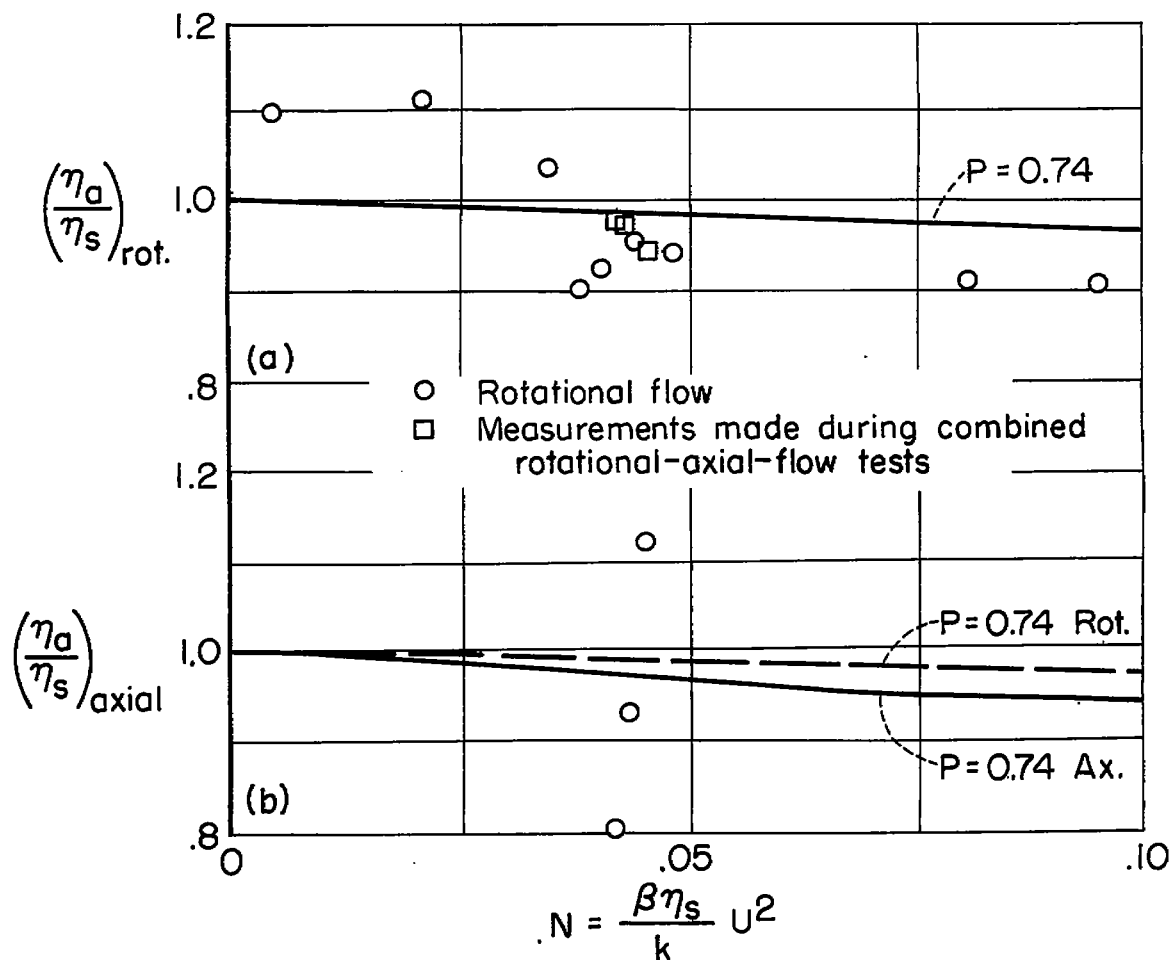


Figure 5.- Relative viscosity results for API 103. Continuous line ($P = 0.74$) based on theory in appendix A.



(a) Rotational-axial-flow results.

(b) Axial-flow results.

Figure 6.- Relative viscosity results for Flowrex 300. Continuous line based on theory in appendix A.

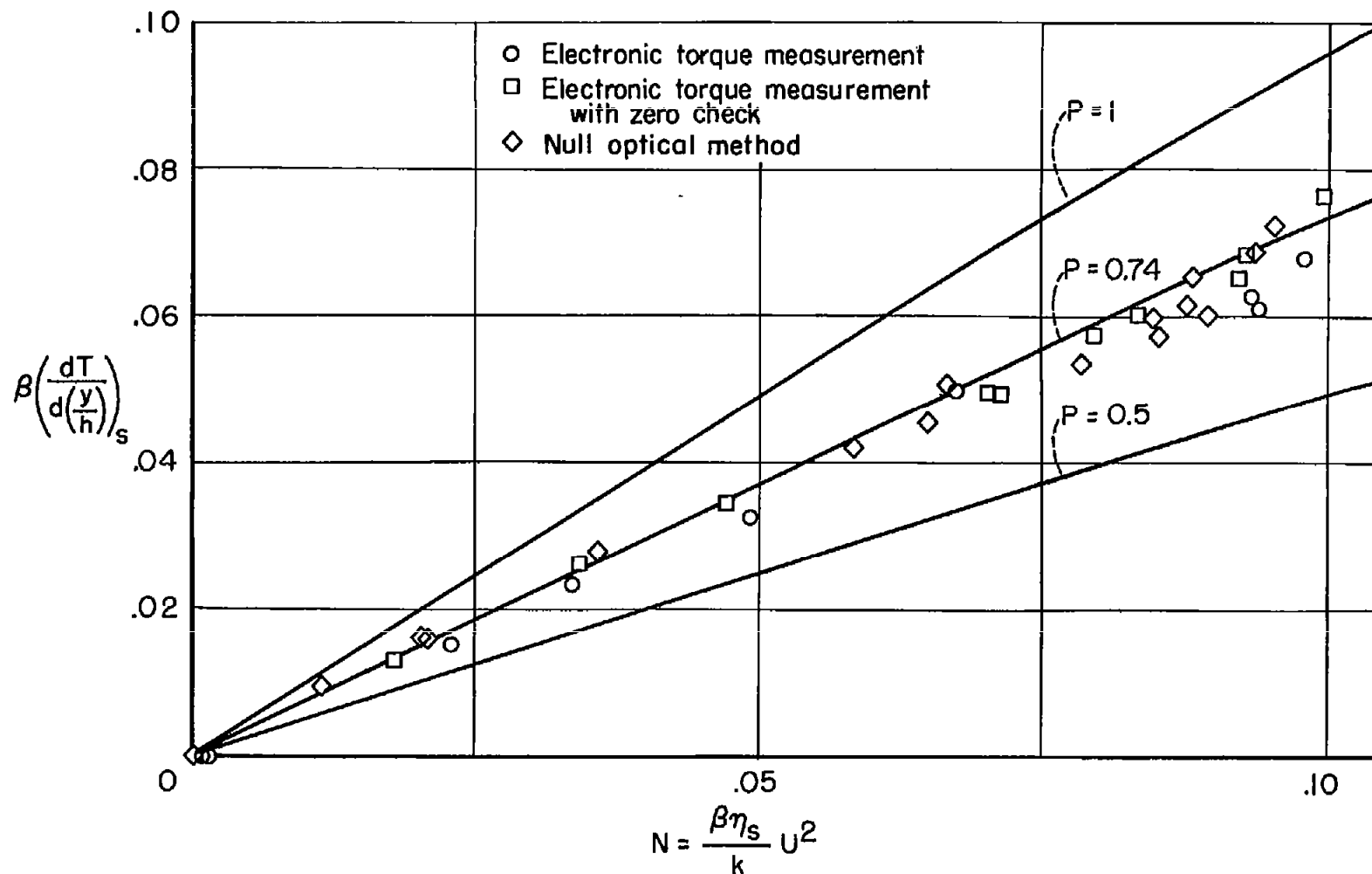


Figure 7.- Temperature-gradient results for Etma Oil Light. Continuous lines based on theory in appendix B. Mean value of P during tests, 0.74.

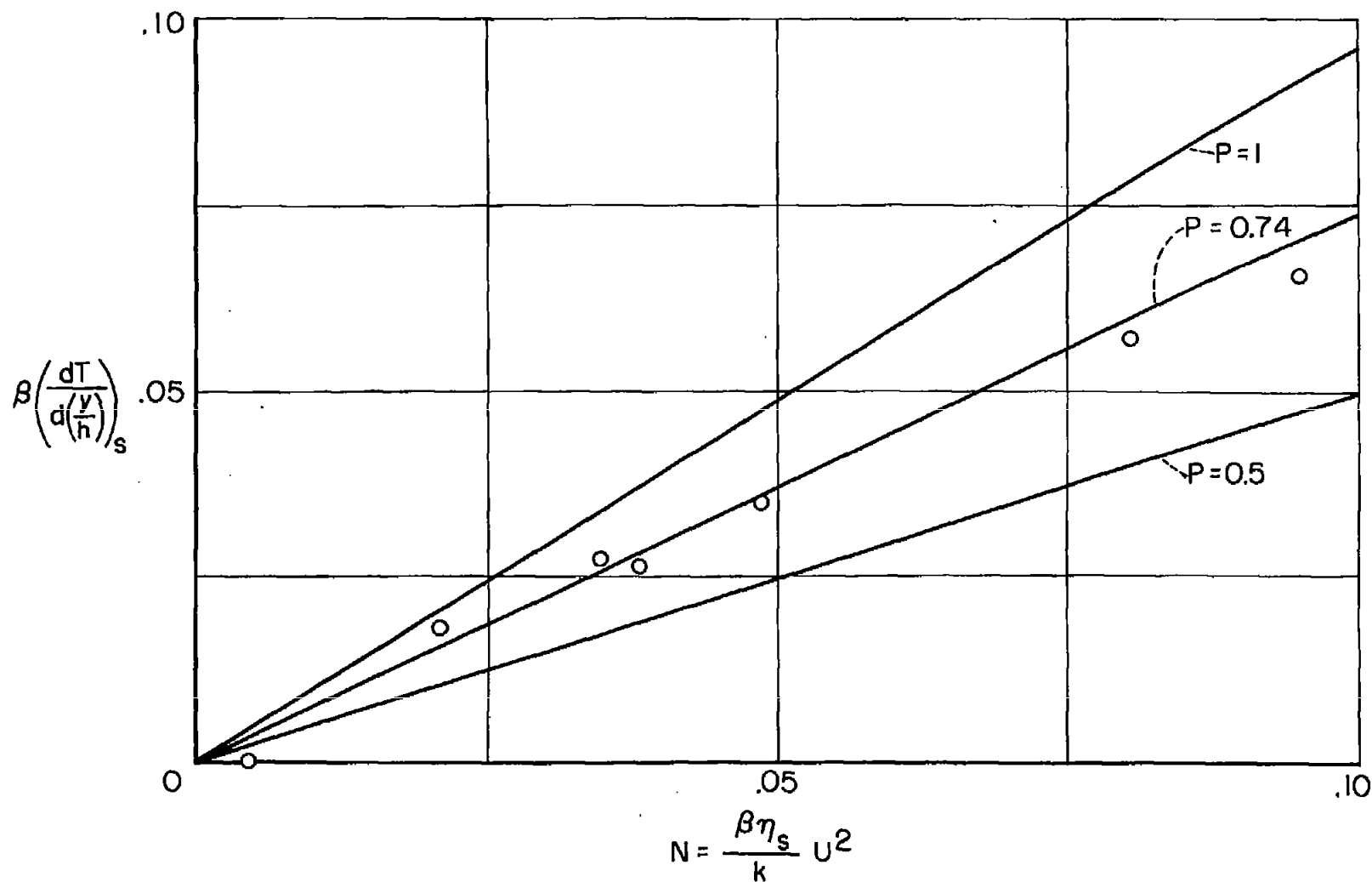


Figure 8.- Temperature-gradient results for Flowrex 300. Continuous lines based on theory in appendix B. Mean value of P during tests, 0.74.

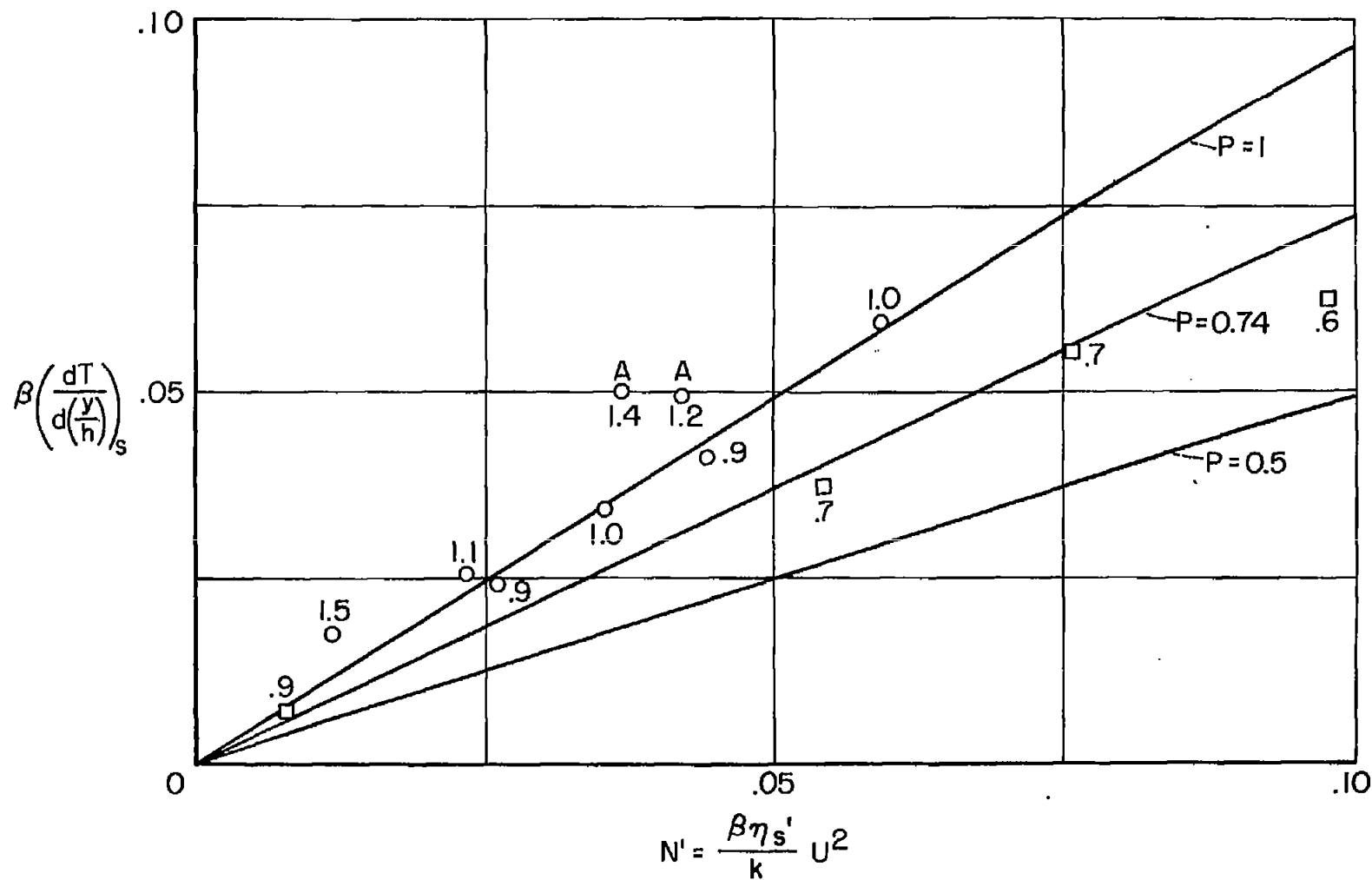


Figure 9.- Temperature-gradient results for API 103. Continuous lines based on theory in appendix B. Individual values of P indicated for each test. Points labeled A indicate anomalous values.